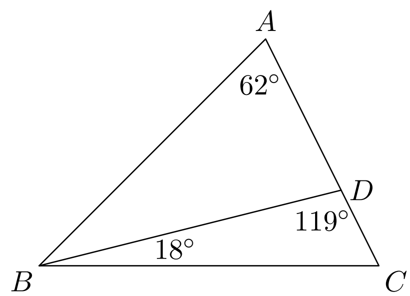
1. We say that numbers a and b are *partners* if $a + b = ab$. Which number is the partner of the number 5?

   (A) $\frac{6}{5}$ (B) $\frac{5}{6}$ (C) $\frac{4}{5}$ (D) $\frac{5}{4}$ (E) 4

2. Angle ABD, in degrees, is



   (A) 110 (B) 57 (C) 80 (D) 137 (E) 43

3. Tonya and Nancy start simultaneously at each end of a marked path on a skating rink, and skate towards each other along the path until they meet. Then, they turn around and skate back along the path. Tonya and Nancy skate at constant speeds, and Tonya skates slightly faster than Nancy. Who returns to her starting point first?

   (A) Tonya (B) Nancy (C) They arrive at the same time
   (D) It depends on how much faster Tonya is (E) It depends on how long the path is

4. A coin with diameter 1/2 is tossed onto a floor which is tiled by squares of side length 1. What is the probability that the coin lands entirely within one of the squares?

   (A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) $\frac{\pi}{4}$ (E) $\frac{4}{\pi^2}$

5. Given that m and n are the roots of the equation $7x^2 + 9x + 21 = 0$, the value of $(m + 7)(n + 7)$ is

   (A) 37 (B) 301 (C) 61 (D) 427 (E) 43

6. A positive integer n is called *fine* if there exists a 5-digit number that contains each of the digits 1, 2, 3, 4, and 5 once, which is divisible by n. For example, 73 is fine because 52341 is divisible by 73. What is the smallest positive integer that is not fine?

   (A) 6 (B) 7 (C) 8 (D) 9 (E) 10

7. Suppose f is a function such that $f(x+1, y) = f(x, y) + y + 1$, $f(x, 0) = x$, and $f(x, y) = f(y, x)$ for all x and y. Then $f(12, 5)$ is equal to

   (A) 77 (B) 60 (C) 17 (D) 83 (E) 67

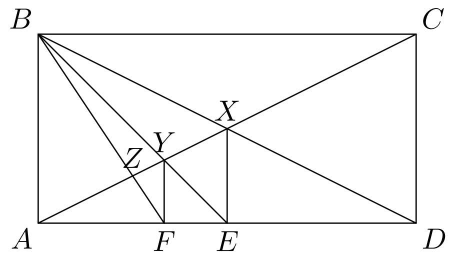
8. Alex writes down all the numbers from 1 to 2010 on a piece of paper. How many times does he write the digit 1?

   (A) 1491 (B) 1492 (C) 1601 (D) 1602 (E) 1611

9. The smallest positive integer such that $\sqrt{n} - \sqrt{n-1} < 0.01$ is

   (A) 2500 (B) 2335 (C) 2501 (D) 2336 (E) 2233

10. Let $ABCD$ be a rectangle. Let AC and BD intersect at X, and let E be the foot of the perpendicular from X to AD. Let AC and BE intersect at Y, and let F be the foot of the perpendicular from Y to AD. Finally, let AC and BF intersect at Z. If $AB = 1$ and $AD = 2$, then ZF is equal to



(A) $\frac{1}{3}$ (B) $\frac{\sqrt{2}}{5}$ (C) $\frac{3}{10}$ (D) $\frac{2\sqrt{5}}{15}$ (E) $\frac{\sqrt{13}}{12}$

11. An acute triangle has side lengths a, b, and c, where $a < b < c$. A side of the triangle is chosen, and the triangle is rotated around the side in space, forming a solid of revolution. Which side should be chosen to form the solid with the greatest volume?

(A) The side with length a (B) The side with length b (C) The side with length c
(D) All solids have the same volume (E) There is not enough information in the problem

12. Let x and y be positive real numbers such that $x^{\log_y x} = 2$ and $y^{\log_x y} = 16$. What is x?

(A) 2 (B) $\sqrt[3]{4}$ (C) $2^{\sqrt{2}}$ (D) $2^{\sqrt[3]{2}}$ (E) $2^{\sqrt[3]{4}}$

13. A *perfect power* is a number of the form m^n, where m and n are positive integers greater than 1. How many positive integers less than or equal to 2^{12} can be expressed as a perfect power?

(A) 72 (B) 81 (C) 85 (D) 87 (E) 97

14. Let n be the ten-digit number 9999999999. The sum of the digits of n^3 is

(A) 99 (B) 108 (C) 180 (D) 199 (E) 297

15. Let

$$x = \frac{2 \cdot 4 \cdot 6 \cdots 2008 \cdot 2010}{1 \cdot 3 \cdot 5 \cdots 2007 \cdot 2009}.$$

Then

(A) $x < 40$ (B) $40 < x < 70$ (C) $70 < x < 100$
(D) $100 < x < 130$ (E) $x > 130$

1. Guy has a collection of 12 rulers, whose lengths are 1, 2, 4, 8, ..., 2048, in meters. He arranges the 12 rulers in a line, in random order. What is the probability that there is a continuous block of rulers whose lengths add up to 2010 meters? (Express your answer in lowest terms.)

2. In triangle ABC, $AB = BC$. Let P be a point on BC such that $AP = 6$, $BP = 2$, and $PC = 4$. Find AC.

3. Find all values of k for which the system of equations

$$\begin{aligned} kx + y + z &= k, \\ x + ky + z &= k, \\ x + y + kz &= k \end{aligned}$$

does *not* have a unique solution in x, y, and z.

4. A particle starts at one vertex of a cube. At each step, the particle moves to an adjacent vertex. How many ways can the particle arrive at the opposite vertex of the cube after five steps? (The particle may visit the opposite vertex before the fifth step, as long as it is at the opposite vertex after the fifth step.)

5. When expressed in decimal, the number 2^{6677} contains 2010 digits. How many digits does the number 5^{6677} contain?

6. Let θ be an angle such that $\sin\theta + \cos\theta = \frac{1}{3}$. Find the value of $\sin^5\theta + \cos^5\theta$, in lowest terms.

7. Let M be the midpoint of side BC in triangle ABC. If $AB = 17$ and $AM = 4$, then find the maximum value of $\cos\angle BAC$, in lowest terms.

8. The function $f(x)$, defined for $0 \le x \le 1$, has the following properties:

   (i) $f(0) = 0$.

   (ii) If $0 \le x < y \le 1$, then $f(x) \le f(y)$.

   (iii) $f(1-x) = 1 - f(x)$ for all $0 \le x \le 1$.

   (iv) $f(\frac{x}{3}) = \frac{f(x)}{2}$ for all $0 \le x \le 1$.

   Express $f(\frac{2}{7})$ in lowest terms.